

## 1. Counting Operations: Image Loops

Images are stored as grids of pixels. The function below paints every pixel in an even-numbered column black.

```
def paint_even_cols(img, height, width):  
    for y in range(height):  
        print("outer loop")  
        for x in range(width):  
            print("inner loop")  
            if x % 2 == 0: # even columns only  
                img.set_rgb(x, y, 0, 0, 0)
```

a. Suppose height = 3 and width = 4. List every (x, y) pair for which `img.set_rgb` gets called.

| y | x values where <code>set_rgb</code> is called |
|---|-----------------------------------------------|
| 0 |                                               |
| 1 |                                               |
| 2 |                                               |

b. For a general image with height =  $h$  rows and width =  $w$  columns, how many times does each print run?

`print("outer loop")` runs \_\_\_\_\_ times.

`print("inner loop")` runs \_\_\_\_\_ times.

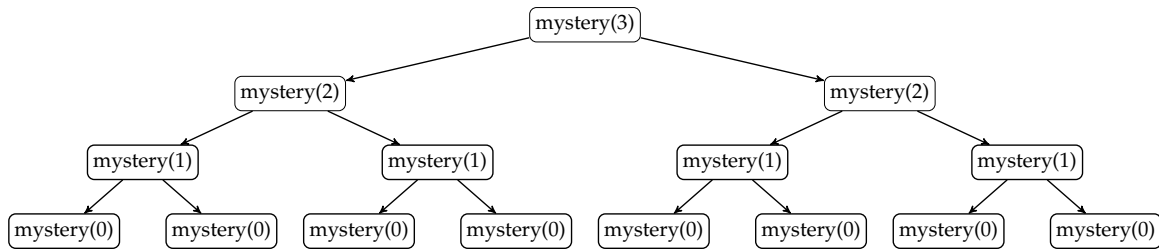
c. Now suppose the image is *square* (height = width =  $n$ ). What is the Big-O runtime?

$O(\text{_____})$

## 2. Recursive Calls

Here is a recursive algorithm. The call tree below shows all the calls made when we run `mystery(3)`.

```
def mystery(n):
    if n == 0:
        return 1
    return mystery(n - 1) + mystery(n - 1)
```



a. Every call to `mystery(n)` (for  $n \geq 1$ ) makes exactly \_\_\_\_\_ recursive calls.

b. What does `mystery` return for small values of  $n$ ?

`mystery(0)` = \_\_\_\_\_    `mystery(1)` = \_\_\_\_\_    `mystery(2)` = \_\_\_\_\_    `mystery(3)` = \_\_\_\_\_

In general, `mystery(n)` returns \_\_\_\_\_

c. Count how many times each call appears in the tree above.

| Call                    | Times called |
|-------------------------|--------------|
| <code>mystery(3)</code> |              |
| <code>mystery(2)</code> |              |
| <code>mystery(1)</code> |              |
| <code>mystery(0)</code> |              |
| <b>Total calls</b>      |              |

d. Complete the table below for larger values of  $n$ .

| $n$ | Total calls to <code>mystery</code> |
|-----|-------------------------------------|
| 0   | 1                                   |
| 1   | 3                                   |
| 2   | 7                                   |
| 3   |                                     |
| 4   |                                     |
| 5   |                                     |

e. Each time  $n$  increases by 1, the total number of calls (circle one):

stays the same    doubles    triples    grows by  $n$

f. Based on your answer to part e, what is the Big-O runtime of `mystery(n)`?

$O(\text{_____})$