

1. Counting Operations

To find the Big-O runtime of a program, we take two steps:

- (1) Count the number of operations, in terms of n .
- (2) Remove all multiplicative constants, and keep only the highest-order term.

Here are some examples of step (2):

$$\begin{array}{llll} 5n + 3 & \rightarrow & O(n) & \text{drop the } +3, \text{ then drop the } 5 \\ 100n + 2 & \rightarrow & O(n) & \text{a big constant is still just a constant} \\ 2n^2 + 50n + 7 & \rightarrow & O(n^2) & n^2 \text{ is the highest-order term} \\ \frac{1}{2}n^2 & \rightarrow & O(n^2) & \text{the constant can be a fraction} \\ 8 & \rightarrow & O(1) & \text{no } n \text{ at all} \end{array}$$

a. Try these two yourself:

$$6n^2 + 300n \rightarrow O(\text{___}) \quad 2.5n \rightarrow O(\text{___}) \quad 9n^3 + 100n^2 + 7n \rightarrow O(\text{___}) \quad 12 \rightarrow O(\text{___})$$

b. Now we will count the operations in a real program. The function below takes a list `lst` of n numbers.

```
def sum_doubles(lst):
    total = 0
    for i in range(len(lst)):
        total = total + lst[i] * 2
    return total
```

How many times does the body of the for loop run, in terms of n ?

c. Count each kind of operation in the whole function. Your answers should be in terms of n . (Do not forget the assignment on the line `total = 0`.)

total = _____ assignments (=) + _____ multiplies (*) + _____ additions (+)

Now add them up:

$$\text{total} = \text{_____} \cdot n + \text{_____}$$

d. Apply step (2) to your answer from part c: drop the multiplicative constant and the lower-order term. What is the Big-O runtime?

$O(\rule{1.5cm}{0.4pt})$

2. Reading Loops

For each program below, first write how many times the print line runs, and then write the Big-O runtime in terms of n where $n = \text{len}(\text{lst})$.

a.

```
for i in range(n):
    for j in range(n):
        print(i, j)
```

print runs $\rule{1.5cm}{0.4pt}$ times.

Runtime: $O(\rule{1.5cm}{0.4pt})$

b.

```
for i in range(n):
    print(i)

for j in range(n):
    print(j)
```

print runs $\rule{1.5cm}{0.4pt}$ times.

Runtime: $O(\rule{1.5cm}{0.4pt})$

c.

```
i = 1
while i < n:
    print(i)
    i = i * 2
```

Hint: write out the values that i takes when $n = 16$:

1, $\rule{1cm}{0.4pt}$, $\rule{1cm}{0.4pt}$, $\rule{1cm}{0.4pt}$, stop

print runs $\rule{1.5cm}{0.4pt}$ times.

Runtime: $O(\rule{1.5cm}{0.4pt})$