

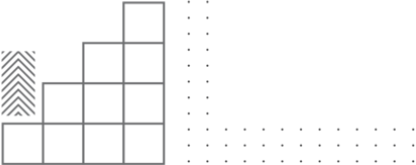
Graph Algorithms: Breadth-First Search (BFS)

Lecture 14B

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AddisCoder 2026

August 13, 2026



Lecture plan

Thursday	Morning	14A	Depth-First Search (DFS)
	Afternoon	14B	Breadth-First Search (BFS) ← we are here
Friday	Morning	15A	Quiz
	Afternoon	15B	Language Models

This morning: depth-first search

- ▶ Go **as deep as possible**,
backtrack when stuck

This morning: depth-first search

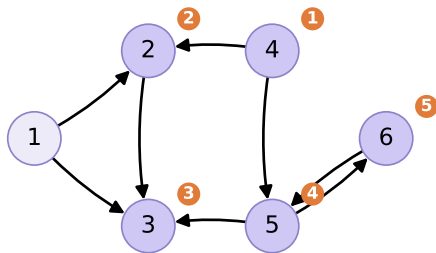
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- ▶ Runs in $O(V + E)$

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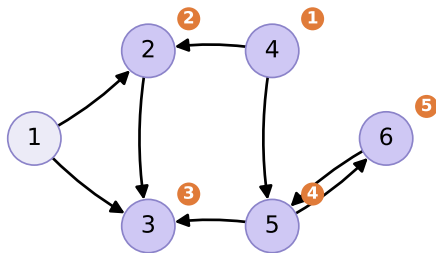
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DFS from 4: 4 2 3 5 6

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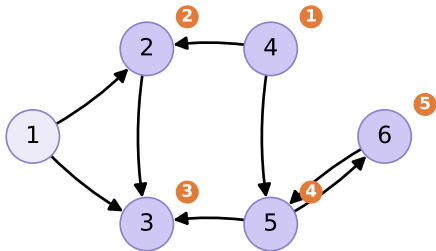
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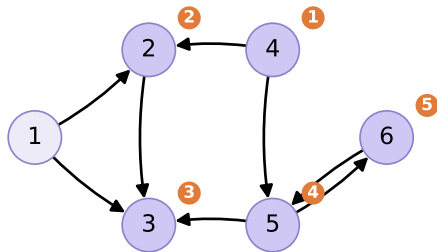
*This afternoon: a search that explores **level by level** instead — almost the same code, **one** data structure changes.*

Is there another sensible order?



DFS from 4: 4 2 3 5 6

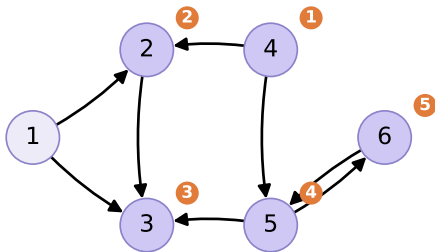
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DFS from 4: 4 2 3 5 6

- DFS went **4** $\rightarrow$ **2** $\rightarrow$ **3**, all the way down, before it ever looked at **5**

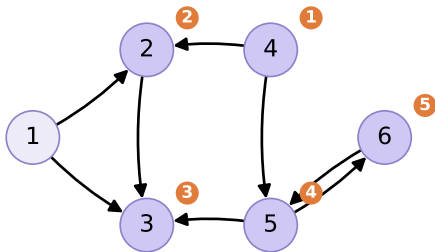
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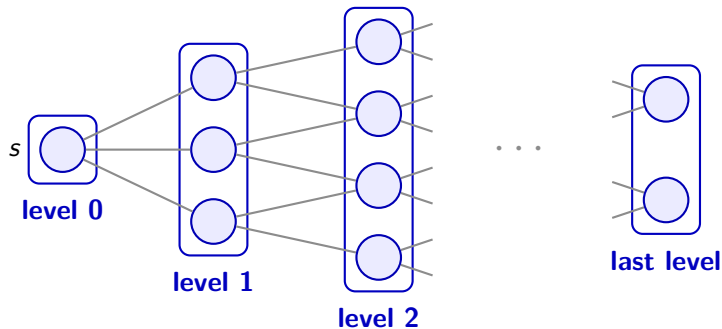
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DFS from 4: 4 2 3 5 6

- ▶ DFS went $4 \rightarrow 2 \rightarrow 3$, all the way down, before it ever looked at **5**
- ▶ But 2 and 5 are both **one step** from 4
- ▶ **Why not visit both of them first?**

Breadth-first search: level by level



*Visit everything **one step** away,
then everything **two steps** away, and so on.*

The BFS algorithm

1. Start at the **starting node** of the graph

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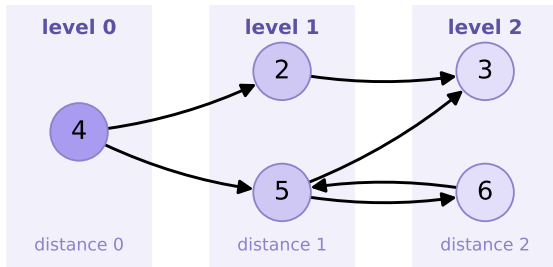
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The BFS algorithm

1. Start at the **starting node** of the graph
 2. Visit all nodes at a particular **level**
 3. After visiting all nodes at that level, move to the **next level**
 4. Repeat until all nodes are visited
- ▶ Just like DFS, we keep a visited dictionary
 - ▶ But to know **what to visit next**, we now need a **queue**

BFS on our graph: what order?

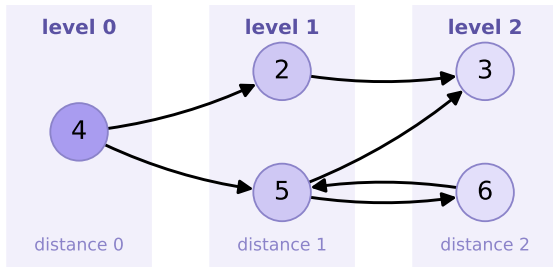


The same graph, redrawn by level. Vertex 1 is not reachable from 4, so it isn't here.

Discuss:

In what order does **BFS** from 4 visit the nodes?

BFS on our graph: what order?



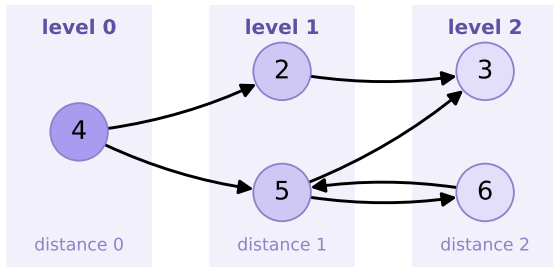
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Discuss:

In what order does **BFS** from 4 visit the nodes?

4 2 5 3 6

DFS said 4 2 3 5 6 — the same nodes, a different order.

What actually has to change?

- ▶ DFS never had a list of “places still to visit”... or did it?

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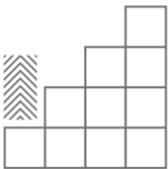
- ▶ DFS never had a list of “places still to visit”... or did it?
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*To go level by level, we need the **opposite**: give back the **oldest** thing first. That data structure is a **queue**.*



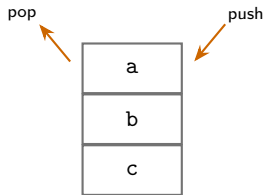
Queues

First in, first out



Stack vs. queue

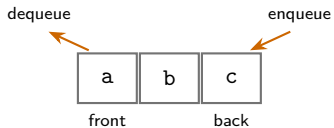
Stack (used by DFS)



Last In, First Out

In and out at the **same** end

Queue (used by BFS)

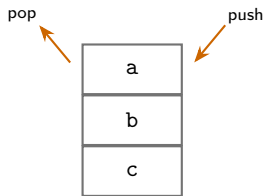


First In, First Out

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Stack vs. queue

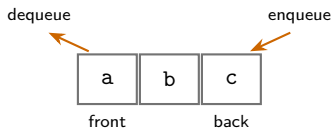
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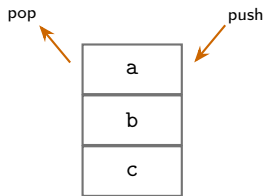
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Discuss: Which one is the queue you **stand in** at a shop?

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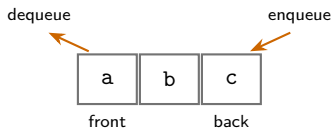
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In and out at the **same** end

Queue (used by BFS)



First In, First Out

In at one end, out at the **other**

Discuss: Which one is the queue you **stand in** at a shop?

And which one is a **pile of plates**?

Queues in Python: deque

```
# In Python, we can represent  
# queues using deque:  
from collections import deque  
  
arr = [5, 7, 1, 8, 2, 10]  
queue = deque(arr)  
print(queue)
```

deque is short for
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print(queue)
queue.append(3)
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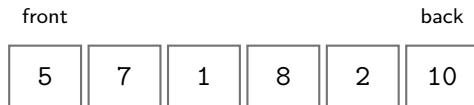
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popleft()
take from the **front**

append(x)
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Discuss: What do the four
print() lines put on
the screen?

A queue, step by step



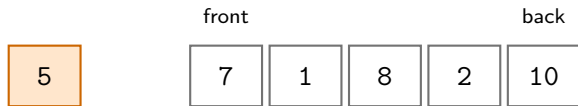
```
queue = deque(arr)  
prints deque([5, 7, 1, 8, 2, 10])
```

A queue, step by step



```
front = queue.popleft()
```

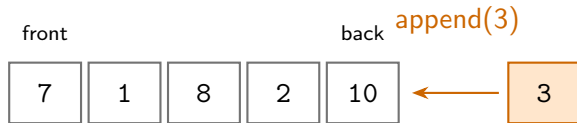
A queue, step by step



returned

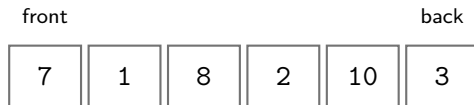
```
print(front)    and    print(queue)
prints 5 then deque([7, 1, 8, 2, 10])
```

A queue, step by step



`queue.append(3)`

A queue, step by step

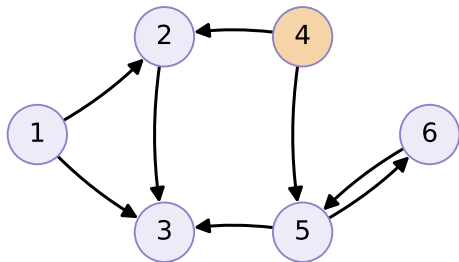


```
print(queue)
```

```
prints deque([7, 1, 8, 2, 10, 3])
```

*Out at the **front**, in at the **back** —
and nothing ever jumps the queue.*

Let's trace it by hand: BFS from 4



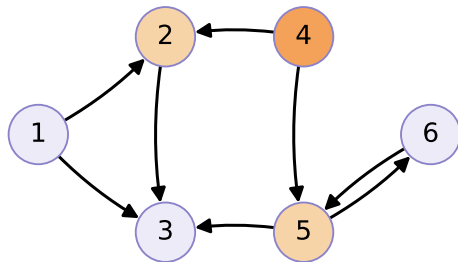
current in queue done not seen

Now: start at 4 — put it in the queue

queue: [4]

visited: {4}

Let's trace it by hand: BFS from 4



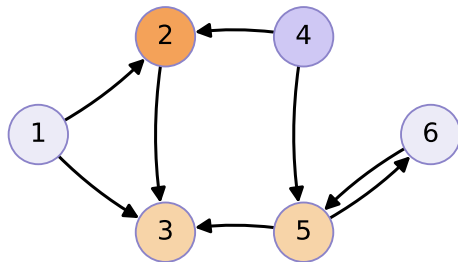
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Now: take **4** — $G[4] = [2, 5]$,
add both

queue: $[2, 5]$

visited: $\{4, 2, 5\}$

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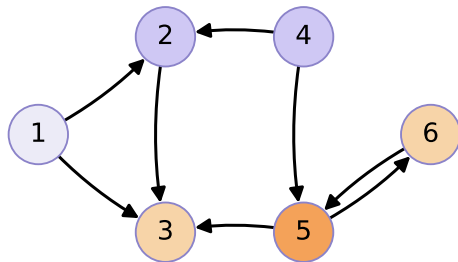
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Now: take **2** — $G[2] = [3]$,
add 3

queue: [5, 3]

visited: {4, 2, 5, 3}

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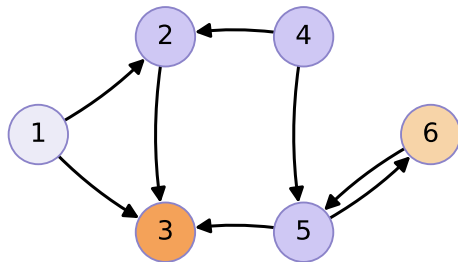
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Now: take **5** — 3 already seen,
add 6

queue: [3, 6]

visited: {4, 2, 5, 3, 6}

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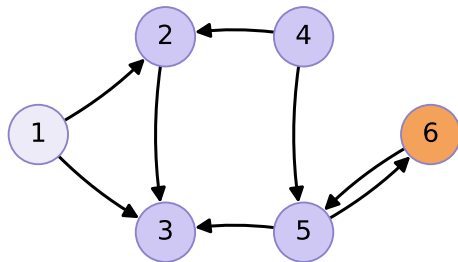
current in queue done not seen

Now: take **3** — $G[3] = []$,
nothing to add

queue: [6]

visited: {4, 2, 5, 3, 6}

Let's trace it by hand: BFS from 4



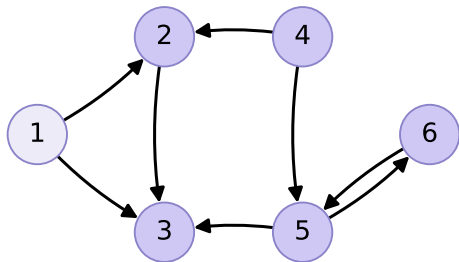
 current  in queue  done  not seen

Now: take **6** — 5 already seen

queue: []

visited: {4, 2, 5, 3, 6}

Let's trace it by hand: BFS from 4



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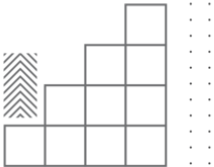
Now: queue empty $\rightarrow$ done!

queue: []

visited: {4, 2, 5, 3, 6}



Let's implement BFS



Reachability again — but breadth-first

```
def connected(source, target, G):
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def connected(source, target, G):  
    visited = {}  
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        for y in G[x]:  
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        if y == target:  
            return True
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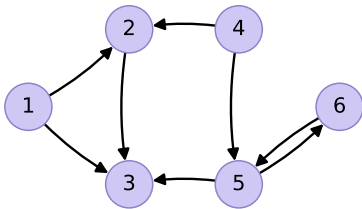
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            if y not in visited:
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    return False
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```

No recursion — the nodes still to visit sit in a queue we can see.

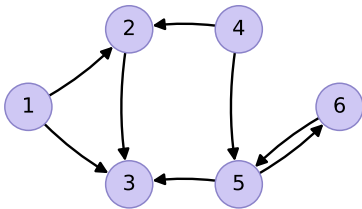
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Discuss:

```
print(connected(1, 5, g_adj))
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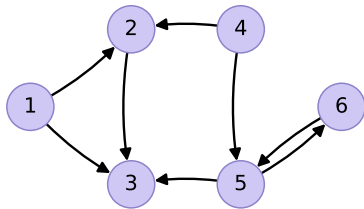


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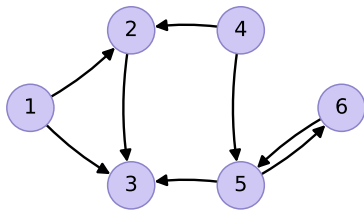
Discuss:

```
print(connected(1, 5, g_adj))
```

False

```
print(connected(4, 6, g_adj))
```

Does it work?



Discuss:

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print(connected(1, 5, g_adj))
```

False

```
print(connected(4, 6, g_adj))
```

True

From searching to traversing

Drop the target, print on the way out — BFS now **visits everything** it can reach:

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        for y in graph[x]:  
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                queue.append(y)
```

Same arguments as `dfs(visited, graph, node)` — swap one for the other in any program.

Running it from every node

```
# g_adj = {1: [2, 3], 2: [3], 3: [],  
#          4: [2, 5], 5: [3, 6], 6: [5]}  
  
for i in g_adj.keys():  
    visited = {}  
    print("BFS from", i)  
    bfs(visited, g_adj, i)
```

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BFS from 1

1

2

3

BFS from 2

2

3

BFS from 3

3

BFS from 4

4

2

5

3

6

BFS from 5

5

3

6

BFS from 6

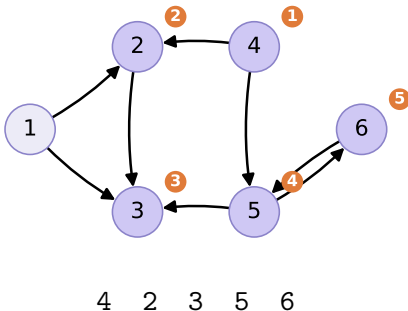
6

5

3

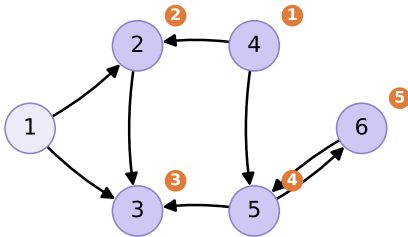
DFS vs. BFS: the same graph, from 4

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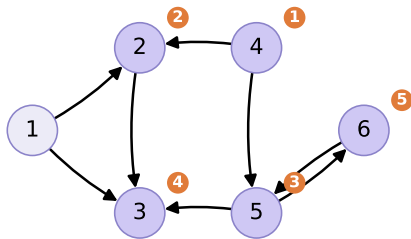
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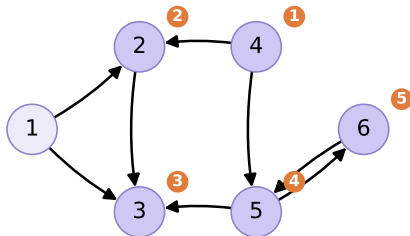
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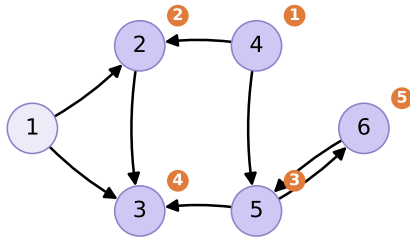
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*Same graph, same five nodes — the **order** is the whole difference.*

One data structure apart

DFS

```
def dfs(visited, graph, node):  
    visited[node] = True  
    print(node)  
    for y in graph[node]:  
        if y not in visited:  
            dfs(visited, graph, y)
```

The **call stack** holds what's left to do — newest first

BFS

```
def bfs(visited, graph, node):  
    queue = deque([node])  
    visited[node] = True  
    while len(queue) > 0:  
        x = queue.popleft()  
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        for y in graph[x]:  
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*DFS lets **recursion** remember what's left — BFS has **no recursion**, just a queue.*

BFS time complexity

Discuss: How many times does BFS put each vertex **into** the queue?
How many times does it **look at** each edge?

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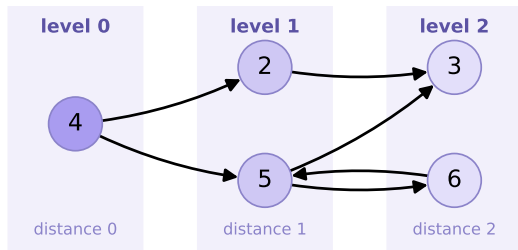
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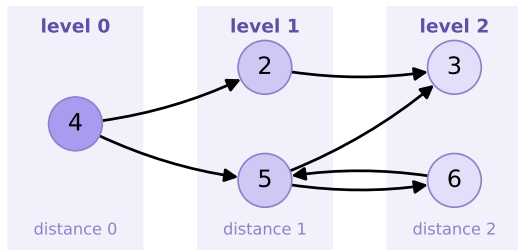
*Exactly the same as DFS. Choosing between them is never about speed
— it is about the **order** you want.*

Why BFS: shortest paths



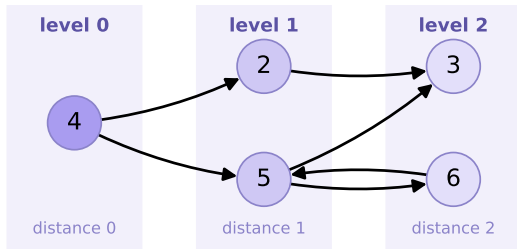
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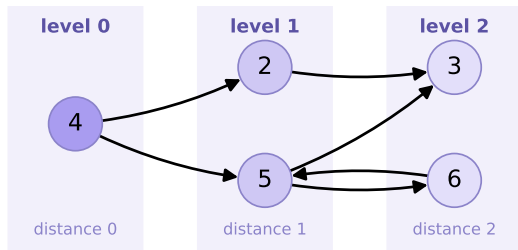
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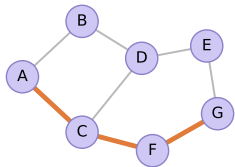


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*BFS finds the **shortest path** (in number of edges) from the start to every node it reaches. DFS does **not**.*

Applications of BFS

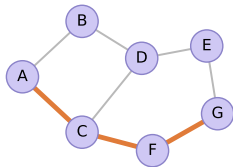
Fewest stops



A to G in 3 stops, not 4

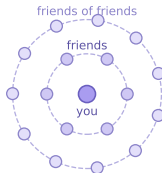
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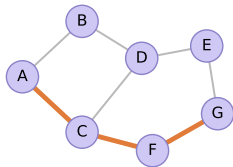
Degrees of separation



Each level: one more "friend of"

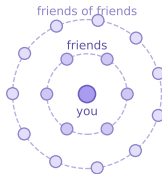
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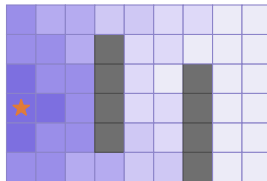
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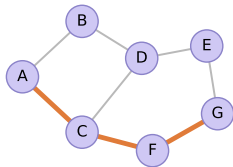
Flood fill



The paint bucket, pixel by pixel

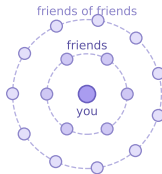
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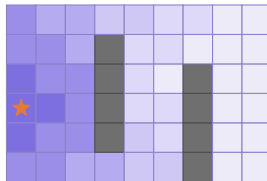
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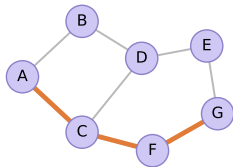


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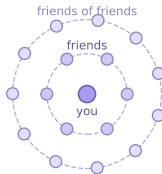
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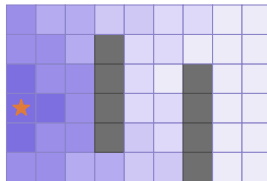
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*Whenever “**how far**” or “**how few steps**” is part of the question, reach for BFS.*

So which one should I use?

	DFS	BFS
Explores	deep along one branch	level by level
To-do list	the call stack — recursion keeps it for you	a deque you keep yourself
Shortest path?	no	yes (fewest edges)
Memory	one <i>path</i> at a time — usually far less	one whole <i>level</i> at a time — can be huge
Finds a target	sooner if it is far away	sooner if it is near
Good for	backtracking; cycles in a <i>di-rected</i> graph	distances, “how few steps”, nearest match

Beyond DFS and BFS

For DFS and BFS, every edge counts the same: **one step**. Change how you pick the *next* node, and you get a whole family:

- ▶ **Dijkstra's algorithm** — edges have *lengths*; always take the **closest** node next (a **priority queue** instead of a queue)

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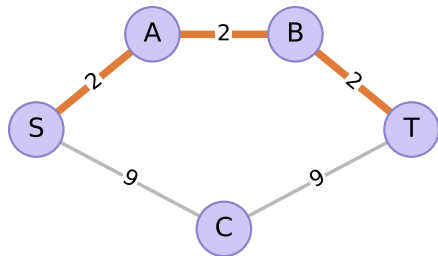
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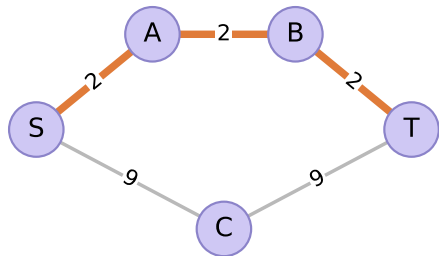
Stack → deep. Queue → level by level. Priority queue → nearest by distance. Same skeleton every time.

Dijkstra's algorithm: when steps have a *length*



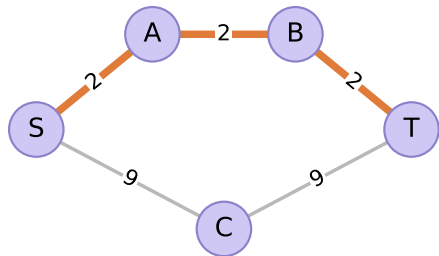
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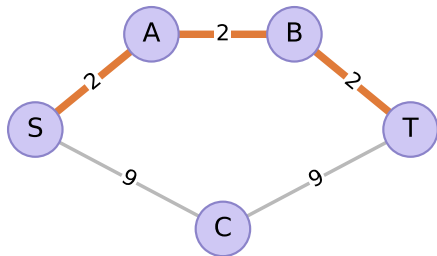
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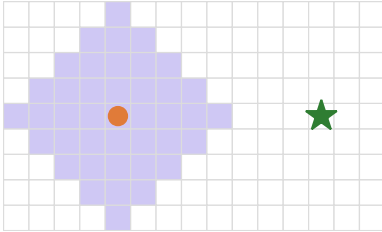


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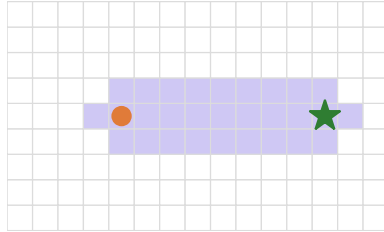
BFS is Dijkstra for the special case where every edge has length 1.

A^* search: point the search at the goal

Dijkstra: spreads out in every direction

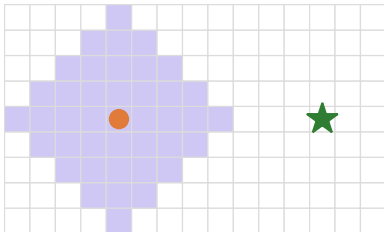


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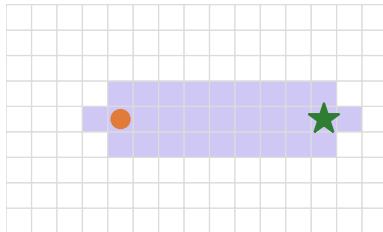


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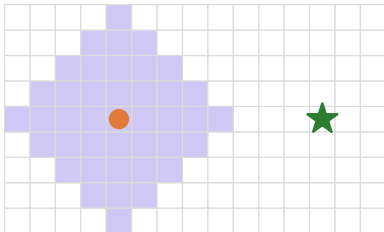
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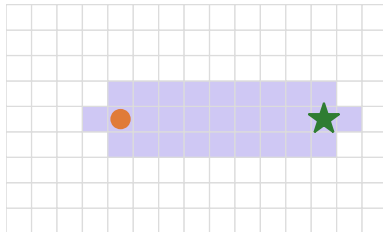
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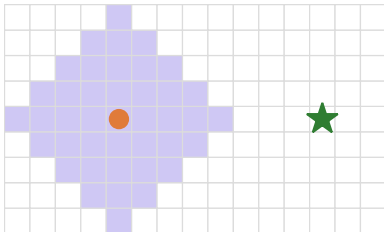
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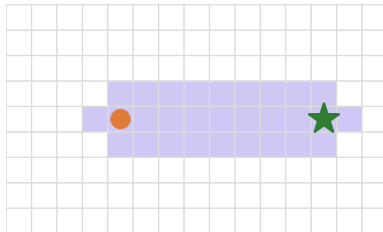
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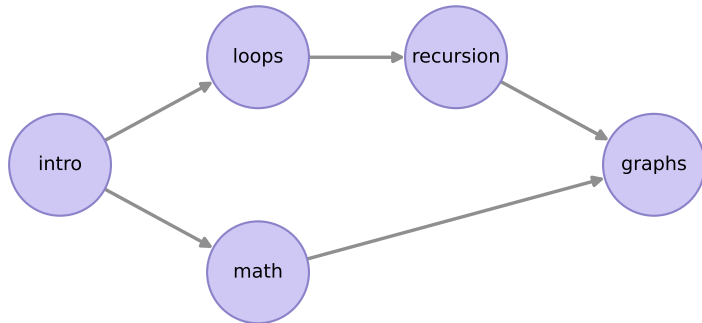
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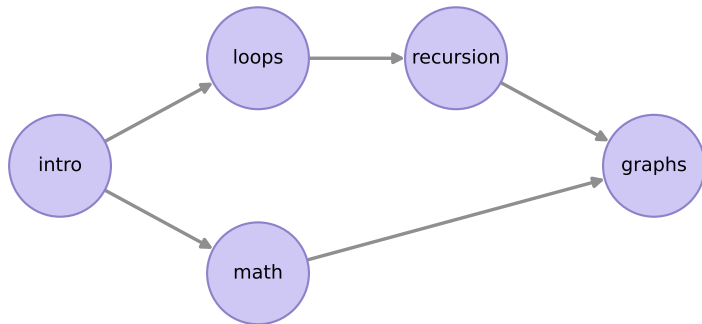
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Same answer, far less searching — this is what a map app is really doing.

Topological sort: what order can I do things in?

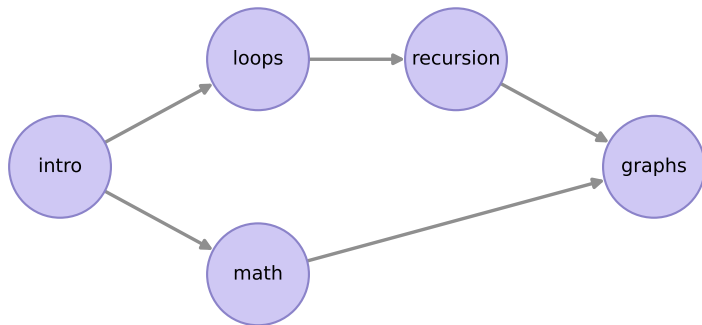


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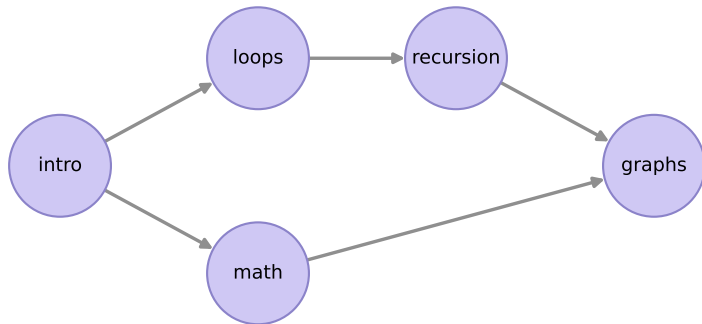
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intro → loops → math → recursion → graphs
- ▶ Built on **DFS** — and it only exists if the graph has **no cycle**

Wrapping up

This afternoon:

- ▶ BFS explores **level by level** — nearest nodes first
- ▶ A **queue** replaces the call stack: `popleft()` instead of recursion
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